the SL model yields unrealizable results in homogeneous shear flow due to a defect in their analysis. In Fig. 2, the time evolution of the invariant function $F \equiv 1 + 9II + 27III$ predicted by each model in homogeneous shear flow is shown for the anisotropic initial conditions: $b_{11} = -0.32$, $b_{22} = b_{33} = 0.16$, $b_{12} = b_{23} = b_{13} = 0$, and $SK/\varepsilon = 15$. For realizable turbulence, we must have $F \geq 0$. It is clear that for these initial conditions, the FLT and SSG models yield realizable solutions whereas the SL model yields unrealizable results! Hence, the primary theoretical constraint by which the SL model was formulated is in error. Furthermore, Durbin and Speziale[16] have shown that any pressure-strain model can be made realizable by a simple readjustment of the Rotta coefficient motivated by a stochastic analysis. Consequently, realizability constraints do not now appear to be a useful means for the basic construction of models.

In conclusion, it must be said that the good performance of a model in these basic turbulent shear flows provides no guarantee that it will do well in more complex turbulent flows. However, these simple test flows do bear directly on how well a model will perform in equilibrium turbulent boundary layers which form a cornerstone for many engineering applications. A model that cannot predict these simple test cases accurately should be abandoned for use in more complex engineering flows.

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Zones in Coflowing Planar Inviscid Supersonic Twin Streams

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Introduction

WITH the advent of a multitude of numerical techniques, the selection of an appropriate method for a given problem has become an extremely tough and time consuming job. At present it is considered to be a matter of experience and intuition. Quantitative selections based on the absolute criteria of speed, accuracy, and ease of implementation are not feasible. However, partially quantitative selections are possible with the knowledge of flow regimes that might be encountered in the flowfield. In many cases this information can be gained quickly and in an inexpensive way by performing computations relating to the topology of the flowfield. This Note demonstrates the usefulness of this methodology.

In dimensions greater than one the number of free variables becomes very large, and generalized parametric solutions and holistic insight into the phenomena occurring in the flowfield are difficult to derive. Because of this, most of the studies providing insight remain confined to one-dimensional analysis only.[1] The topological flow analysis also seems to be one of the best alternatives for such studies. Furthermore, certain assumptions, like that of equal static pressures of both the streams downstream of splitter plate,[2] are not easily justified except in an intuitive way. However, topological flow analysis can render semiquantitative justifications of these assumptions.

In the literature, few such studies have been performed to increase the understanding of the mixing of a compressible subsonic stream with a supersonic stream.[1] However (within the scope of our knowledge), topological information for the mixing of two supersonic streams is lacking. Thus, in this article, a topological analysis of the flowfield of two coflowing planar supersonic streams, issuing from the same or different stagnation chambers, that come into contact downstream of an infinitely thin splitter plate is carried out. The simplifying assumptions made for the ease of analysis are the steady, inviscid, adiabatic flow of calorically perfect gas. The nature of the problem demands a numerical algorithm that gives sharp zonal demarcation. Hence the method of characteristics along with isentropic flow and shock relations has been used.

Preview of the Flowfield

The flowfield is very sensitive to inlet flow parameters. Slight variations in these result in different flow patterns. Downstream of the nozzle exit plane, a shock wave and an expansion fan are formed at the splitter plate's end.[3] The shock appears in the stream having low static pressure (stream 1) at the nozzle exit plane and the expansion fan in the stream having high static pressure (stream 2). If the flow after the shock is supersonic, it reflects from the wall. The type of reflection (regular or Mach) is governed by the inlet flow parameters. The reflected shock intersects the slip stream, and if its downstream flow is supersonic, the shock gets reflected as an expansion wave. From the point of intersection a compression front is transmitted into stream 2, which may be a shock or an isentropic compression wave depending on the inlet flow parameters and the minimum entropy production principle. These waves propagate downstream, reflecting from the walls and the slip stream, forming a complex wave dominated flowfield and lead to a Mach disk or a shock-like compression front (secondary

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shock) in one of the streams. The location of the this front in a particular stream is governed by the inlet parameters.

Dimensional analysis of the flowfield reveals that its geometry is a function of eight inlet parameters: inlet heights H_1 and H_2, and inlet flow parameters M_1, M_2, P_1 (or P_{01}), P_2 (or P_{02}), γ_1, and γ_2.

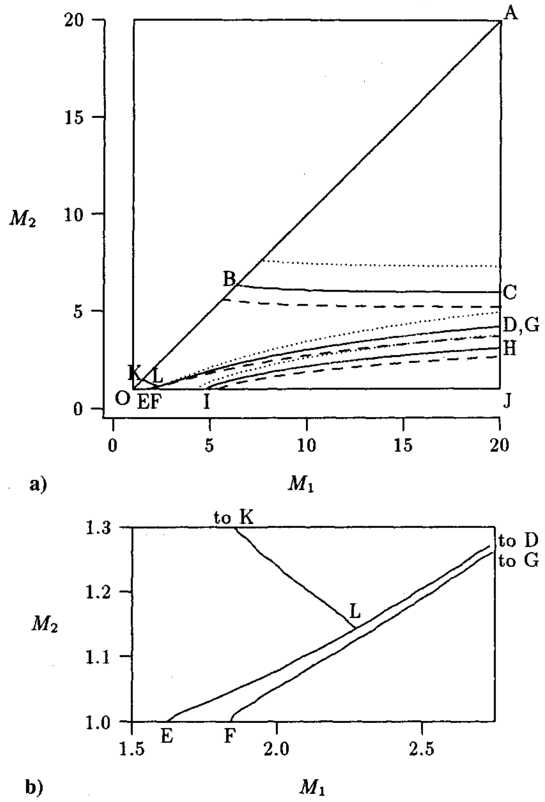

Fig. 1 Topology of the flowfield and effect of change in γ; b) is enlargement of that section of a) for $\gamma_1 = \gamma_2 = 1.4$; for all the curves $P_{01} = P_{02}$ and $H_1 = H_2$. Line styles indicate $\gamma_1 = \gamma_2 = 1.3$ (······), 1.4 (——), and 1.5 (- - - -).

This shows that the most generalized treatment of the spatial variation of a flow parameter Σ (e.g., P, T, ρ, etc.) can be concluded only in a 10-dimensional parameter space wherein Σ (X, Y; H_1, H_2; M_1, M_2, P_1, P_2,γ_1, γ_2) is determined. Thus, some of the important representative graphs with fixed or free variations in one or more of the inlet parameters have been presented.

Results

Topology of the Flowfield

Regions of qualitatively identical flow pattern in the M_1M_2 plane are shown in Fig. 1. For all the stream Mach number combinations, H_1/H_2 is such that the abscissa of the point of intersection of the reflected oblique shock and the slip stream (X_{SS}) is less than that of the first expansion ray from stream 2 and the slip stream (X_{ES}). The figure is symmetric about the line OA and hence only the lower half is shown. In this figure, the shock is formed in stream 1 and the Mach number combination (M_1, M_2) lies below OA. (M_1, M_2) lying above OA implies that the stream with the shock is labeled stream 2. If (M_1, M_2) lies on OA, the static pressures of both the streams at the splitter plate's end are equal, the shock and the expansion fan are not formed, and the slip stream remains straight.

If the (M_1, M_2) combination lies in region ABC or HIJ, then the flow downstream of the shock at the splitter plate's end is subsonic. In region $CBOIH$ the downstream flow is supersonic. Mach reflection of the incident oblique shock occurs in region $GFIH$, leading to complicated mixed subsonic and supersonic flow downstream of the reflected oblique shock. In region $CBOFG$ only regular reflections occur. The flow downstream of the reflected oblique shock is supersonic in region $CBOED$ and subsonic in $DEFG$, which is an extremely narrow region. The border lines of this region, ED and FG, approach each other and tend to meet as inlet stream Mach numbers increase.

Line styles in Fig. 1 show the effect of change in γ_1 and γ_2 on regions of identical flow pattern. As γ_1 and γ_2 increase, border lines between various regions move forward on the M_1 axis and down on the M_2 axis, narrowing down all the regions except ABC whose size increases. The maximum value of γ (5/3 for monatomic gases) places a limit on the minimum size of various regions. Since $\gamma = 1$

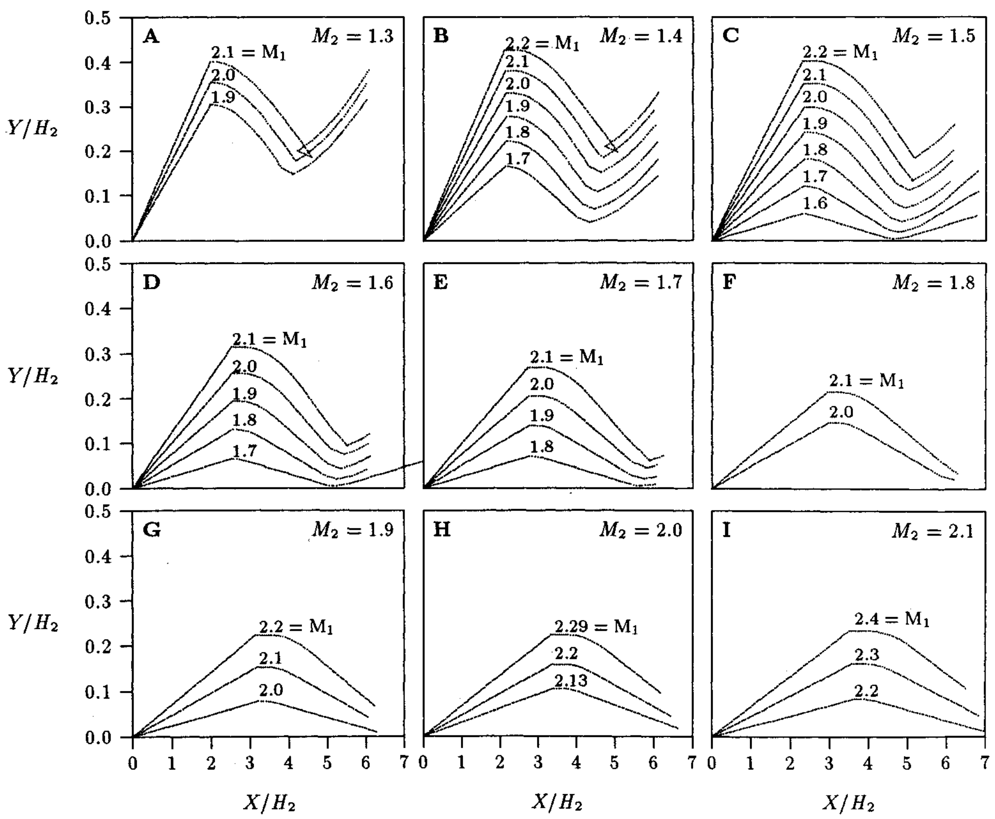

Fig. 2 Computed slip-stream shapes; for all the curves $P_{01} = P_{02}$, $H_1 = H_2$, and $\gamma_1 = \gamma_2 = 1.4$.

can be achieved only for an ∞-atomic gas, it behaves as a singularity during the solution procedure and the graphs of Fig. 1 become undefined at this value. However, it can be seen from Fig. 1 that as γ_1 and γ_2 decrease to 1, points B and C move toward A. From this it can be inferred that in the limiting case of $\gamma \rightarrow 1$, B and C will be located at (∞, ∞). Similarly, D, G, and H will be located at (∞, ∞) and E, F, and I will be located at (1, 1).

Under the prestated constraint on H_1/H_2, the region of flow that can be analyzed with the method of characteristics is *CBOED* in Fig. 1. On violation of this constraint expansion waves from stream 2 are transmitted into stream 1 upstream of the reflected oblique shock, curving it and increasing its strength. As a result many streams of continuously varying Mach number are formed downstream of the reflected oblique shock, which may be subsonic. This phenomenon introduces a demarcation line (*KL*, for $H_1/H_2 = 1$ and $\gamma_1 = \gamma_2 = 1.4$) in the region *CBOED* and reduces the domain of applicability of analysis (to *CBKLD*). The exact location and shape of this line depends on H_1/H_2, γ_1, and γ_2. In the region *KOEL*, $X_{SS} > X_{ES}$ and in *CBKLD*, $X_{SS} < X_{ES}$. Though *CBKLD* appears to be an open region from the upper end *CD*, it probably is not so. For certain inlet Mach number combinations an upper limit might also be imposed by formation of an oblique shock in stream 2 from the point of intersection of the reflected oblique shock and the slip stream. The location and shape of this line will also be functions of the tunnel geometry and inlet flow parameters.

Shape of the Slip Stream

Computed slip-stream shapes are shown in Fig. 2. The discontinuity in the slope of the slip stream at the point of intersection by the reflected oblique shock is clearly marked. Furthermore, for a given value of M_2, the change in the abscissa of the point of intersection of the reflected oblique shock and the slip stream is negligibly small. This point houses the first major maxima of the shape. The ordinates of the maxima increase as M_1 increases and have an upper bound of ≈ 0.45 before the occurrence of the secondary shock in stream 2 upstream of the following minima. Downstream of the maxima, curvature of the slip stream slowly increases, and a point of inflection occurs just upstream of the first minima. The abscissa of the minima is almost twice that of the maxima. The probability of occurrence of a shock in the upstream neighbourhood of the minima is very high, as is indicated by the kinks in the slip-stream shape at this location for high values of M_1. The kinks indicate actual formation of Mach disk or shock-like compression front in stream 2 because of the intersection of Mach waves of the same family. Increase in the size of the kink with an increase in M_1 indicates its proportionality with the strength of the secondary shock. The absence of a kink at this location does not imply the complete absence of shocks. In fact, it indicates the formation of shocks in stream 1 downstream of the minima rather than in stream 2. It should also be observed that the slip stream flattens out as M_2 increases. The maxima and the minima move forward on the abscissa, and the ordinates of these points decrease. Still, the ratio of the abscissa of these points remains almost the same.

All the computations were performed in single precision to an accuracy of 10^{-5} except in Fig. 1 where it was reduced to 10^{-2} in M_1 and M_2 to save computer time. For computation of the slip-stream shape, the angular separation between consecutive characteristics was taken to be 1 deg.

Discussion

Though the methodology presented in the preceding sections is promising in that it can reveal vital structural information about a flow space, it should be used with caution since assumptions of inviscid and planar flowfield break down in most practical situations. Viscosity leads to dissipation in the strength of shock waves, changing the analysis drastically. Still, with appropriate modifications in the numerical scheme the methodology outlined here can be adopted in most flow situations. It is expected that even after adaptations are incorporated to take care of the viscosity, three-dimensionality, and mixing in the flowfield, the philosophy, efficiency, and use of the analysis methodology presented here would remain unaltered.

The slip stream has been considered as an infinitely thin interface, which is acceptable as a first approximation. In actual flowfields the slip stream is considerably thick because of mixing, and thickness increases as one goes downstream. In these cases the infinitely thin interface may (perhaps) be taken to be some mean line of the slip stream.

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Suppression of Vortex Asymmetry and Side Force on a Circular Cone

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Introduction

It is a well-documented fact that the flow about a slender pointed body of revolution at zero yaw becomes asymmetric at some high angle of attack; e.g., see Ref. 1. The initially symmetric vortex pair on the lee side rearranges into an asymmetric configuration with the asymmetry starting either at the tail of a long afterbody or at the tip of a slender, pointed nose. This asymmetric flow leads to a side force acting on the body. A similar phenomenon of an initially symmetrical vortex pair becoming asymmetric has been observed long ago on a circular, two-dimensional cylinder set into motion impulsively in a fluid initially at rest.[2] This flow was studied theoretically by Foeppl.[3] He investigated the stability of the symmetrical vortex pair with respect to small, symmetric and antisymmetric displacements and found that the symmetrical vortex pair is stable for symmetric but unstable for antisymmetric disturbances. This result suggested that a "fin" between the vortices would lead to stable, symmetric flow. These considerations were carried over by means of unsteady-flow analogy to the three-dimensional vortex flow behind the corresponding inclined cylinder and qualitatively to the inclined cone.[4] The effect of such a fin on the flow past a slender cone was studied by means of flow visualization in a water tunnel at a low value of Reynolds number and in a low-speed wind tunnel at about 10 times the value of *Re*. The fin largely suppressed vortex-flow asymmetry; therefore, a quantitative investigation of the surface pressures and forces on the cone without and with fin was carried out.

Results were reported by Ng,[5] concurrently with Stahl's first results,[4] on such a fin on a nose suppressing vortex asymmetry. Various other methods have been found to be effective in reducing or suppressing vortex-flow asymmetry; e.g., see Ref. 1.

Experimental Setup and Techniques

The wind tunnel that we used at King Fahd University of Petroleum and Minerals (KFUPM) is of the partially open return type with a closed horizontal test section of 0.8 m × 1.1 m and length of

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